

DEPARTMENT OF MATHEMATICS & STATISTICS

Basic Business Statistics 1A –Tutorial, 2011

SECTION 1: BASIC MATHEMATICS

1. Evaluate on your calculator
 - a. $(2.3+4.6+9.2+17.3)/4$
 - b. $28^{0.5}$
 - c. 0.2^{-3}
 - d. $e^{1.6}$
 - e. $e^{-1.6}$
 - f. $4!, 1!, 6!, (-3)!, 8!$

2. For the data $x: 2, 3, 5, 1, 4, 3, 2, 4$ find

a. $\sum_{i=1}^n x_i$

b. \bar{x}

c. $(\sum_{i=1}^n x_i)^2$

d. $\sum_{i=1}^n x_i^2$

e. $\sum_{i=1}^n (x_i - \bar{x})$

f. $\sum_{i=1}^n (x_i - \bar{x})^2$

g. $\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n}$

3. Repeat question 2 for the data

$x: 2, 3, 4, 6, 1, 3, 7, 2, 2, 3$

SECTION 2 : TYPES OF DATA, LEVELS OF MEASUREMENTS

1. Define the following:

- a) Statistic
- b) Parameter
- c) Variable
- d) Sample
- e) Population
- f) Census
- g) Experiment

2. Discuss the following :

- a) levels of data measurement.
- b) types of variables
- c) data types
- d) methods of data collection

- 3 State with reasons whether the following data are continuous or discrete.
- The temperature of the sea in summer.
 - The number of children in schools
 - The lifetime of a light bulb.

4 What is the difference between primary data and secondary data?

5 Distinguish between qualitative and quantitative data.

State whether each of the following is quantitative or qualitative data. If it is quantitative, state whether it is discrete or continuous.

- Your favourite chocolate
 - The number of times you visited the cinema in the last 6 months.
 - Your HIV status.
 - The weight of your luggage on your last flight.
 - The sex of your child.
- 6 Postal questionnaires are often used to gather data if the targeted geographical area is large. List 5 advantages and 5 disadvantages of postal questionnaires.
- 7 Discuss the advantages and disadvantages of the following methods of gathering data.
- Personal interviews.
 - Telephone interview.
 - Experimentation.
- 8 Classify each of the following as nominal, ordinal, interval or ratio level data.
- The time required to produce each tyre on an assembly line.
 - The number of pints of milk a family drinks in a month.
 - The ranking of four machines in your plant after they have been designated as excellent, good, satisfactory and poor.
 - The telephone area code of clients in Namibia.
 - The age of each student in your BBS class.
 - The dollar sales at your local pizza place each month.
 - The Polytechnic of Namibia student identity number.
 - The response time of City Policy to local emergencies.

SECTION 3: PRESENTATION OF STATISTICAL DATA

1. The Namibian Marketing Journal summarized the buying behaviour of a thousand families. The result was to categorize the families into 5 buying categories A, B, C, D, and E respectively. The following table shows data that was collected from 50 families.

A	C	E	B	D	C	D	B	D	C
D	B	D	E	C	A	D	C	D	E
D	C	A	B	D	C	B	E	C	D
B	C	D	C	D	C	E	A	D	C
C	B	D	D	B	D	C	B	B	A

- Construct the frequency distribution for the qualitative data in the table. Describe the information that is discerned from the frequency distribution table about the categories of the buyers, in terms of numbers of buyers in each category.
- Construct the relative frequency distribution for the dataset.
- Construct the bar charts for (i) the frequency distribution and (ii) the relative frequency distribution for the dataset.
- Construct a pie chart for the relative frequency distribution for the dataset.

2. The following table summarizes the number of days taken by 100 customers to pay what they owe to a certain company.

36	35	19	44	35	36	24	20	27	24
15	24	42	26	31	26	17	30	17	28
33	37	28	34	45	30	35	25	45	30
20	31	30	36	39	31	36	34	37	33
34	27	22	31	23	13	43	18	42	18
38	19	40	25	29	45	31	29	32	23
28	32	28	32	30	31	29	28	29	41
30	41	32	29	33	46	44	28	21	22
52	42	14	29	23	37	38	21	37	33
32	29	35	35	24	26			60	45

- a. Construct the frequency distribution for the dataset.
 - b. Construct the relative frequency distribution for the dataset.
 - c. Construct the cumulative frequency distribution for the dataset.
3. A property developer analyzed the amount of office space available in 40 office blocks in the CBD of Gobabis. The following floor areas were recorded.

175	150	178	144	182	172	126	143
146	116	182	140	162	155	176	140
165	165	142	165	168	120	160	155
170	162	177	164	158	118	135	162
195	124	135	150	178	190	160	185

- a. Is the data
 - I. Continuous or discrete
 - II. Quantitative or qualitative.
 - III. Nominal, ordinal, ratio or interval scaled?
 - b. Determine the range of the data.
 - c. Use Sturges Rule to determine k, the number of classes, also determine the class width and the appropriate class limits.
 - d. Construct a frequency distribution for the data.
 - e. Construct a histogram for the data. Interpret your histogram.
 - f. Construct a frequency polygon and interpret it.
 - g. Construct a less than ogive for the data.
 - h. Construct a more than ogive for the data. From your graph what percentage of office blocks has available office space in excess of 175square metres?
 - i. Plot the less than and more than ogives on the same axis and determine the median.
 - I. What percentage of office blocks has less than 145 square metres floor space.
 - II. What office space area is provided by the lower 70% of office blocks.
 - III. What % of office blocks provides more than 140 square metres of office space.
4. The following data are marks of students in an examination.

24	61	36	81	45	37	78	65	60	39
47	51	35	82	63	60	58	37	52	24
90	87	18	8	29	46	35	64	27	50

- a. Construct a frequency distribution for the data.
- b. Compile the cumulative frequency table and draw the cumulative frequency curve.
- c. If the minimum for grade A was fixed at 74, estimate from your curve the percentage of candidates obtaining grade A.

5. The following data represent the costs of a sample of 30 postal mailings (in N\$) by a company.

3.67	2.75	5.47	4.65	3.32	2.09
1.83	10.94	1.93	3.89	7.2	2.78
3.34	7.8	3.2	3.21	3.55	3.53
3.64	4.95	5.42	8.64	4.84	4.10
9.15	3.45	5.11	1.97	2.84	4.15

a) Using dollars as a stem and cents as a leaf construct a stem and leaf plot of the data. What does the stem and leaf tell you about the costs?

6. A company deployed 50 salespersons in a marketing zone. The following table summarizes the number of hours each salesperson used over a period of one month.

128	203	40	149	84	164	187	124	144	130
62	93	128	87	155	94	157	195	110	133
141	104	69	121	113	79	162	149	204	103
143	30	42	164	71	105	151	161	165	175
148	114	116	122	146	197	167	184	156	140

- Construct a less than and more than ogive on the same graph. Estimate the middle quartile.
- From the grouped data, calculate the mean, variance and standard deviation.
- Calculate the co-efficient of variation and interpret the results.

7. The following data are the average weekly mortgage interest rates for a 60-week period.

7.29	7.03	7.14	6.77	6.35
6.69	7.02	7.40	7.16	6.96
6.98	7.56	6.75	6.87	7.11
7.39	7.28	6.97	6.90	6.57
7.11	6.95	7.23	7.31	7.00
7.30	7.17	6.96	6.78	7.30
7.16	6.78	6.79	7.07	7.03
6.87	6.80	7.10	7.13	6.95
7.08	7.24	7.34	7.47	7.31
6.96	6.70	6.57	6.88	6.84
7.02	7.40	7.12	7.16	7.16
6.99	6.94	7.29	7.05	6.84

- Construct a frequency distribution for this data.
- Calculate and display the class midpoints, relative frequencies, and cumulative frequencies for this frequency distribution.

SECTION 4: DESCRIPTIVE STATISTICS

1. The ratio of actual costs to estimated costs for 49 new product developments in a pharmaceutical firm are given in the following distribution:

Cost ratio	Number of products
0.00-<0.50	1
0.50-<1.00	5
1.00-<1.50	18
1.50-<2.00	6
2.00-<2.50	6
2.50-<3.00	10
3.00-<3.50	3

- Calculate the mean, median and modal cost ratio for this distribution.
- Calculate the variance and standard deviation for this distribution.
- Calculate the co-efficient of variation and interpret the results.

2. The human resource department of a company analyzed the level of absenteeism of 56 employees who reported ill over the past year.

Days absent	Number of employees
3 - < 7	14
8 - < 12	22
13 - < 17	11
18 - < 22	6
23 - < 27	33

- Find the mean median and modal level of absenteeism.
 - Is the distribution of days absent symmetrical?
 - Calculate the variance and standard deviation for this distribution.
 - Interpret the standard deviation.
 - Calculate the co-efficient of variation and interpret the results.
3. The following data represents the percentages of family income allocated to groceries for a sample of 50 shoppers

% of family income	No of shoppers
10 - < 20	6
20 - < 30	14
30 - < 40	16
40 - < 50	11
50 - < 60	3

- Calculate the mean mode and median for the distribution.
- Which of the three measures of central location values would you publish in a consumer magazine as being representative of the actual percentage of family income spend on groceries? Explain.
- Calculate the variance & standard deviation for this distribution.

SECTION 5: PROBABILITY THEORY

- Specify a sample space S for each of the following random experiments.
 - The results of 3 flips of a coin are observed.
 - The marital status of a loan applicant is solicited.
 - Two six-sided dice are tossed and the sum of the spots turning up is noted.
 - The number of customers served by a restaurant on a particular day recorded.
 - After 20 shoppers are asked if they are satisfied with parking accessibility, the number of positive responses is noted.
- An aerospace company has submitted bids on two separate government contracts A and B. The company feels that it has 50% chance of winning contract A and 40% chance of winning contract B. Furthermore it believes that winning contract A is independent of winning contract B.
 - What is the probability that the company will win both contracts?
 - What is the probability that the company will win at least 1 of the contracts?
- Suppose the aerospace company in the question above feels that it has a 60% chance of winning contract A and 30% chance of winning contract B. Given that it wins contract B, the company believes it has an 80% chance of winning contract A.
 - What is the probability that the company will win both contracts?
 - What is the probability that the company will win at least 1 of the contracts?
 - If the company wins contract B, what is the probability that it will not win contract A?
- A certain city has one morning newspaper and one evening newspaper. It is estimated that 20% of the city's households subscribe to the morning newspaper and 60% subscribe to the evening newspaper. Of those who subscribe to the morning newspaper, 80% also subscribe to the evening paper. What proportion of the households
 - Subscribes to both papers.
 - Subscribes to at most one of the papers.
 - Subscribes to neither paper.
- A union's executive conducted a survey of its members to determine what the members felt were the important issues to be discussed during the upcoming negotiations with management. 74% felt job security was an important

issue while 65% felt pension benefits were an important issue. Of those who felt pension benefits were an important issue, 60% also felt job security was important.

- a. What % of the members felt that both issues were important?
- b. What % felt at least one of the issues was important?

SECTION 6: CONDITIONAL PROBABILITY

6. The trustees of a company’s pension plan have solicited the employees’ feelings towards a proposed revision in the plan. A breakdown of the responses is shown in the table below.

Decision	Blue collar workers	White collar workers	Managers
For	67	32	11
Against	63	18	9

Suppose that an employee is selected at random, find the probability that the employee selected is:

- a. A blue collar worker.
 - b. Against the proposed revision.
 - c. Not a manager.
7. The following table shows the 300 employees of a small manufacturing company cross-classified on the basis of age and work category.

Age Group	Work category			Total
	Production	Sales	Office	
<25	50	2	50	102
25-40	70	24	50	144
>40	40	4	10	54
Total	160	30	110	300

An employee is selected at random from this population. Calculate the probability that the employee is:

- a. Under 40 years of age.
 - b. A production worker.
 - c. A sales person and between 25 and 40 years of age.
 - d. Over 40 given that he/she is an office worker.
 - e. A production worker or under 25 or both.
8. A wine dealer has classified the last 200 customers according to the criteria given in the following table:

Type of wine bought	Age of customer			Total
	Under 30	30-50	50 and above	
South African	100	30	20	150
French	2	2	16	20
German	2	16	2	20
Other	4	6	0	10
Total	108	54	38	200

Find the following probabilities.

- a. $P(\text{age} < 30)$.
 - b. $P(\text{South African})$.
 - c. $P(\text{South African or French})$.
 - d. $P(\text{age} < 30 \text{ and South African})$.
 - e. $P(\text{age} < 30 \text{ or South African})$.
9. Given the following probabilities about 2 events E_1 and E_2 :
- $P(E_2/E_1)=0.30$.
 - $P(E_1)=0.60$.
 - $P(E_2)=0.40$.

Find $P(E_1 \text{ and } E_2)$.

10. A company has 1000 credit customers. They are classified according to the size of their balances and the timeliness of their payments.

Last payment	Balance			Total
	<100	100-500	>500	
On time		0.45		0.85
Late			0.03	
Total	0.20		0.30	

- Copy the table and fill in the missing probabilities.
 - If a customer is selected at random, what is the probability that his last payment was late?
 - If a customer is selected at random, what is the probability that the balance is less than 100?
 - If the balance is less than 100, what is the probability that the last payment was late?
11. A candidate writes two examinations A and B. His chance of passing examination A is 0.3 and that of passing examination B is 0.4 and that of passing both is 0.1. What is the probability that the candidate passes examination A or B.
12. Sixty percent of the population of a town read magazine A or magazine B and 10% read both. If 50% read magazine A what is the probability that a person selected at random reads magazine B?
13. Of a company's employees, 30% are women and 6% are married women. Suppose an employee is selected at random. If the employee selected is a woman, find the probability that she is married.

SECTION 7: TOTAL PROBABILITY AND BAYES THEOREM.

14. An electronics plant, it is known from past experience that the probability is 0.84 that a new worker who attended the company's training programme will meet the production quota, and that the corresponding probability is 0.49 for a new worker who has not attended the company's training programme. If 70% of all new workers attend the training programme, what is the probability that a new worker will meet the production quota?
15. In a certain community, 8 percent of all adults over 50 have diabetes. If a health service in this community correctly diagnoses 95 percent of all persons with diabetes as having the disease and incorrectly diagnoses 2 percent of all persons without diabetes as having the disease, find the probabilities that
- The community health service will diagnose an adult over 50 as having diabetes.
 - A person over 50 diagnosed by the health service as having diabetes actually has the disease.
16. In a manufacturing company, a batch of 50 units was manufactured by three production sections in the following compositions:- 18 units were manufactured in the first production, 20 in the second and 12 in the third. The probability that a unit from section one or two meets the requirements is 0.9 while that for a unit from section 3 is 0.6. Determine the probability that a unit picked for inspection meets acceptable quality requirements.
17. Three clerks in a bank branch attended a group of customers. The first clerk attended 25% of the customers, the second 35% and the third 40%. Of the customers attended by the first clerk, 5% were not satisfied and of those attended by the second clerk, 4% were dissatisfied while for the third clerk 3% were dissatisfied. The branch manager randomly picked one customer who was dissatisfied. What is the probability that he was served by the third clerk?
18. A company manufactures a total of 8000 motorcycles a month in three plants A, B and C. Of the 8000 motorcycles, plant A manufactures 4000, and plant B manufactures 3000. At plant A, 85 out of 100 motorcycles are of standard quality or better. At plant B, 65 out of 100 motorcycles are of standard quality or better and at plant C, 60 out of 100 motorcycles are of standard quality or better. The quality controller randomly selects a motorcycle and finds it to be of sub-standard quality. Calculate the probability that it has come from plant B.

SECTION 8: RANDOM VARIABLES AND EXPECTATION

19. The number of accidents that occur annually on a busy stretch of highway is a random variable.
- What are the possible values of the random variable?
 - Are the values countable? Explain.
 - Is there a finite number of values?
 - Is the random variable discrete or continuous?

20. The distance a car travels on a tank of gasoline is a random variable.
- What are the possible values of this random variable?
 - Are the values countable?
 - Is there a finite number of values.
 - Is the variable discrete or continuous?

21. Consider a random variable with the following distribution.

x	-4	0	1	2
$P(x)$	0.2	0.3	0.4	0.1

Find the following probabilities.

- a). $P(x > 0)$ b). $P(X \geq 0)$ c). $P(0 \leq X \leq 1)$ d). $P(X = -4)$ e). $P(X = -2)$.

Find μ and σ for the random variable. Find also $E(X^2)$ and also $E(3X^2 + 2)$

22. Consider the random variable with the following probability distribution.

$$P(X) = 0.1X \text{ where } X = 1, 2, 3, 4.$$

Express the probability distribution in tabular form and use it to find the following probabilities.

- a). $P(X \geq 1)$ b). $P(X > 1)$ c). $P(2 \leq X \leq 3)$ d). $P(X = 4)$ e). $P(X = 3.5)$

Find the mean and the variance of the random variable X.

23. Let X be the number of dots that turn up when a six sided die is tossed.

- a. Express the probability distribution of X in tabular form.

24. Let X be the number of heads obtained when a fair coin is flipped thrice. Express the probability distribution of X in tabular form.

25. Let X be the random variable with the following probability distribution.

x	5	10	15	20	25
$P(x)$	0.05	0.3	0.25	0.25	0.15

- Find the mean and standard deviation of X.
- Find the mean and standard deviation of 2X.
- Find the expected value and variance of $Y = 4X - 3$.

26. You are planning a journey to Mars. You are told that the mean daytime temperature at that time of the year is -10 degrees Celsius. With a standard deviation of 3 degrees Celsius. You are only familiar with the Fahrenheit scale. The relationship between the two scales is represented by the formula.

$$F = \left(\frac{5}{9}\right)C + 32. \text{ Find the mean and standard deviation of daytime Fahrenheit scale temperatures.}$$

27. Suppose you and a friend have contributed equally to a portfolio of \$10 000 invested in a risky venture. The income X that will be earned on this portfolio over the next year has the following probability distribution.

X	\$500	\$100	\$2000
$P(X)$	0.5	0.3	0.2

- Determine the expected value and variance of the income earned on this portfolio.
- Determine the expected value and variance of your share (one half) of the income.

SECTION 9: DISCRETE PROBABILITY DISTRIBUTIONS

28. Solve the following problems by using the Binomial formula.

- If $n = 4$ and $p = 0.10$, find $P(X = 3)$.
- If $n = 7$ and $p = 0.80$, find $P(X = 4)$.
- If $n = 10$ and $p = 0.60$, find $P(X \geq 3)$.
- If $n = 12$ and $p = 0.45$, find $P(3 \leq X \leq 5)$.

29. Solve for the mean and standard deviation of the following Binomial distributions.
- $n = 20$ and $p = 0.70$
 - $n = 70$ and $p = 0.35$
 - $n = 100$ and $p = 0.50$
30. Find the following values by using the Poisson formula
- $P(X = 5 | \theta = 2.3)$
 - $P(X = 2 | \theta = 3.9)$
 - $P(X \leq 3 | \theta = 4.1)$
 - $P(X = 0 | \theta = 2.7)$
 - $P(X = 1 | \theta = 5.4)$
 - $P(4 < X < 8 | \theta = 4.4)$
31. A shoe store's records show that 30% of the customers purchase by credit card. This morning 20 customers purchased shoes from the store.
- Find the probability that at least 12 of the customers used a credit card.
 - What is the probability that at least 3 customers but not more than 6 used a credit card?
 - What is the expected number of customers using a credit card?
32. a. A discrete random variable can be described by the Binomial distribution if it satisfies four conditions. Briefly discuss each of these conditions.
- b. According to an article in the February 1991 issue of Reader's Digest, patients face a 1 in 20 chance of acquiring an infection while hospitalized. If the records of eight randomly selected hospitalized patients are examined, find the probability that none of the eight have acquired an infection whilst hospitalized.
33. A sign on the gas pumps of a certain chain of gasoline stations encourages customers to have their oil checked claiming that one out of every four cars should have its oil topped up.
- What is the probability that exactly 3 of the next ten cars entering the station need an oil top-up.
 - What is the probability that at least half of the ten cars need a top-up?
34. The number of accidents that occur on an assembly line have a Poisson distribution with an average of three accidents per week.
- Find the probability that a particular week will be accident free.
 - Find the probability that at least three accidents will occur in a week.
 - Find the probability that exactly five accidents will occur in a week.
 - If the accidents in different weeks are independent of each other, find the expected number of accidents to occur in a year.
35. Customers are known to arrive at an exclusive boutique randomly, with an average of two customers arriving per hour.
- What is the probability that more than 3 customers will require service during a particular hour?
 - What is the probability that fewer than 4 customers will require service during a 4 hour period in the morning on a particular day?
36. During the summer months (June to August, inclusive) an average 5 marriages per month take place in a small city. Assuming marriages occur randomly and independently, find the probability of the following
- Fewer than 4 marriages take place in a given summer month.
 - At least 14 but not more than 18 marriages will take place in the whole summer period.
 - Exactly 10 marriages will take place in the two months of July and August.

SECTION 10: CONTINUOUS PROBABILITY DISTRIBUTIONS

37. Use the normal tables to find the following probabilities.
- a). $P(Z \geq 1.7)$ b). $P(Z \geq -0.95)$ c). $P(Z \leq -1.96)$ d). $P(Z \geq -1.96)$ e). $P(Z \leq 2.43)$ f). $P(-2.97 \leq Z \leq 1.38)$ g.) $P(-1.14 \leq Z \leq 1.55)$
38. Let X be a random variable with mean 50 and standard deviation 8. Find the following probabilities.
- a). $P(X \geq 52)$ b). $P(X < 40)$ c). $P(X > 40)$ d). $P(35 \leq X \leq 64)$

39. Given that X follows a normal distribution with mean 64 and standard deviation 0.5 find:
 a). $P(X < 63)$ b). $P(X > 63.7)$ c). $P(62.9 < X < 64.3)$ d). $P(X > x) = 0.1026$ e). $P(X > x) = 0.9772$
40. State three properties of the normal distribution function.
41. Electronic books (e-books) are becoming a reality thanks to the computer age. However, e-books are not cheap as the average price in 2009 was N\$400 per book. Suppose the price of e-books is normally distributed with a standard deviation of N\$150
 What is the percentage of e-books being sold in 2009 were priced:
 a. Between N\$300 and N\$600?
 b. Less than N\$250?
42. The time required to assemble an electronic component is normally distributed with mean 12 minutes and variance 2.25 minutes. Find the probability that a particular assembly takes the following length of time.
 a. More than 14 minutes.
 b. Less than 8 minutes
 c. Between 10 and 15 minutes.
43. Weekly purchases of petrol at a garage are normally distributed with a mean of 5000 litres and a standard deviation of 2000 litres. What is the probability that in a given week, the purchases will be:
 a. Between 2500 and 5000 litres
 b. More than 3760 litres.
44. The lifetime of a certain brand of tyres is normally distributed with a mean of 45000km and a standard deviation of 2500km. The tyres carry a warranty of 40000km.
 a. What proportion will fail before the warranty expires?
 b. What proportion will fail after the warranty expires but before reaching 41000 km?
45. A recent survey by a local municipality established that daily water usage is normally distributed with a mean of 200 litres and a variance of 2000 litres. What is the most amount of water likely to be used daily by the lowest consuming 10 % of households?
46. Six hundred candidates wrote an entrance test for admission to a management course. The marks obtained by the candidates were found to be normally distributed with a mean of 132 marks and a standard deviation of 18 marks. If the top 60 performers were given confirmed admission, what is the minimum mark above which a candidate would be sure of being admitted?
47. The lifetime of a certain make of electric appliance is normally distributed with a mean of 2.5 years and standard deviation of 1.1 years. If the manufacturer of appliance wants to ensure that no more than 3% of the machines will be replaced within the guarantee period, what new guarantee period should they choose?
48. In an intelligence test administered to 1000 children, the average score was 42 and the standard deviation 24. Assuming normal distribution, find:
 a. The number of children with scores between 30 and 60
 b. The score exceeded by the top 100 children.
49. The mean inside diameter of a sample of 200 washers produced by a machine is 10.0 mm and the standard deviation is 0.1 mm. The purpose for which the washer is intended allows for diameters between 9.9 mm and 10.1 mm; otherwise the washers are considered defective.
 Assuming the diameters to be normally distributed, calculate:
 a. The percentage of defective washers produced by the machine.
 b. The number of washers that fall into the largest 10% of the washers.
50. The percentage of protein in a certain brand of dog food is normally distributed with a mean of 11.2% and standard deviation of 0.6%. The manufacturer would like to state on the package that the product has a protein content of at least $x_1\%$ and no more than $x_2\%$. It wants the statements to be true for 99% of the packages sold. Determine the values of x_1 and x_2